SULIT

POLITEKNIK MALAYSIA

BAHAGIAN PEPERIKSAAN DAN PENILAIAN
JABATAN PENDIDIKAN POLITEKNIK
KEMENTERIAN PENDIDIKAN MALAYSIA

JABATAN MATEMATIK, SAINS DAN KOMPUTER

PEPERIKSAAN AKHIR

SESI DISEMBER 2014

BA601: ENGINEERING MATHEMATICS 5

TARIKH : 06 APRIL 2015
MASA : 2.30 PM - 4.30 PM (2 JAM)

Kertas ini mengandungi **EMPAT BELAS (14)** halaman bercetak.
Bahagian A: Struktur (2 soalan)
Bahagian B: Struktur (2 soalan)
Bahagian C: Struktur (2 soalan)
Dokumen sokongan yang disertakan : Formula

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIARAHKAN

(CLO yang tertera hanya sebagai rujukan)

SULIT

SECTION A

BAHAGIAN A

INSTRUCTION:
This section consists of **TWO (2)** questions with 25 marks each. Answer **ONE (1)** question from each part, and **ONE (1)** remaining question from either part A/B/C

ARAHAN :
*Bahagian ini mengandungi **DUA (2)** soalan dengan jumlah 25 markah setiap soalan. Jawab SATU (1) soalan dari setiap bahagian, dan SATU (1) soalan selebihnya dari mana-mana bahagian samada A/B/C.*

QUESTION 1

SOALAN 1

CLO1
C2

(a) Find the value for each of the following by using the definition/formula of hyperbolic functions:

Cari nilai bagi setiap yang berikut dengan menggunakan definisi/formula fungsi hiperbolik.

i. $\cosh\sqrt{8}$

[2 marks]
[*2 markah*]

ii. $\tanh(-2.5)$

[2 marks]
[*2 markah*]

iii. $\sinh^{-1}(\ln 2)$

[2 marks]
[*2 markah*]

iv. $\text{cosech}^{-1}(-3)$

[2 marks]
[*2 markah*]

CLO1
C3

(b) Prove that

Buktikan bahawa

i. $\cosh x - \sinh x = e^{-x}$

[3 marks]
[*3 markah*]

ii. $\cosh(A+B) = \cosh A \cosh B + \sinh A \sinh B$

[6 marks]
[*6 markah*]

CLO1
C2

(c) Sketch a quadrant graph and find the principal value for the following functions:

Lakarkan graf sukuan dan dapatkan nilai utama bagi fungsi-fungsi berikut:

i. $\cos^{-1}(-0.35)$

[4 marks]
[*4 markah*]

ii. $\tan^{-1}(1.192)$

[4 marks]
[*4 markah*]

QUESTION 2

SOALAN 2

CLO1
C3

(a) Find the value for each of the following by using the definition/formula of hyperbolic functions:

Cari nilai bagi setiap fungsi yang berikut dengan menggunakan definisi/formula fungsi hiperbolik:

v. $\sinh(\ln 5)$

[2 marks]
[*2 markah*]

vi. $\operatorname{sech}^{-1}(0.72)$

[2 marks]
[*2 markah*]

vii. $\cosh^{-1}\left(\frac{3}{2}\right)+\coth\left(\frac{5}{4}\right)$

[4 marks]
[*4 markah*]

CLO1
C2

(b) Complete the table below for equation $y = 2\sinh\frac{2x}{3}$. Then sketch the graph in the range of $-4 \le x \le 4$.

Lengkapkan jadual di bawah bagi persamaan $y = 2\sinh\frac{2x}{3}$. *Seterusnya lakarkan graf pada julat* $-4 \le x \le 4$.

x	-4	-2	0	2	4
y					

[5 marks]
[*5 markah*]

CLO1
C3

(c) Given that $\cosh x - \sinh x = me^{-x}$, find the value of m.

Diberi $\cosh x - \sinh x = me^{-x}$, *cari nilai bagi m.*

[4 marks]
[*4 markah*]

CLO1
C3

(d) Prove that $\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$

Buktikan $\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$

[8 marks]
[*8 markah*]

SECTION B
BAHAGIAN B

INSTRUCTION:
This section consists of **TWO (2)** questions with 25 marks each. Answer **ONE (1)** question from each part, and **ONE (1)** remaining question from either part A/B/C

ARAHAN :
*Bahagian ini mengandungi **DUA (2)** soalan dengan jumlah 25 markah setiap soalan. Jawab SATU (1) soalan dari setiap bahagian, dan SATU (1) soalan selebihnya dari mana-mana bahagian samada A/B/C.*

QUESTION 3
SOALAN 3

CLO2
C3

(a) Differentiate the following functions with respect to x.

Bezakan setiap fungsi yang berikut terhadap x.

i. $y = \tanh^{-1}\left(\frac{x}{4}\right)$

[3 marks]
[*3 markah*]

ii. $y = e^{\ln(\cosh x)}$

[4 marks]
[4 *markah*]

iii. $y = \frac{\cos^{-1}(3x)}{2x}$

[5 marks]
[5 *markah*]

CLO2
C3

(b) Determine $\frac{dy}{dx}$ for the implicit function $\cosh x^2 + \ln y = x^2 y^3 + 5$

Tentukan $\frac{dy}{dx}$ *untuk fungsi tersirat* $\cosh x^2 + \ln y = x^2 y^3 + 5$

[7 marks]
[*7 markah*]

CLO2
C3

(c) Given $z = 4y\cosh(3x) + 3x\sinh(2y)$, find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial y^2}, \frac{\partial^2 z}{\partial x \partial y}$ and $\frac{\partial^2 z}{\partial y \partial x}$.

Diberi $z = 4y\text{kosh}(3x) + 3x\sinh(2y)$, *cari* $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial y^2}, \frac{\partial^2 z}{\partial x \partial y}$ and $\frac{\partial^2 z}{\partial y \partial x}$.

[6 marks]
[*6 markah*]

QUESTION 4
SOALAN 4

CLO2
C3

(a) Integrate the following functions with respect to x.

Kamirkan fungsi-fungsi berikut terhadap x.

i. $\int \sinh(9-7x)dx$

[3 marks]
[*3 markah*]

ii. $\int 5\cosh^3 x \sinh x \, dx$

[5 marks]
[*5 markah*]

iii. $\int \frac{3}{7-(x+2)^2}dx$

[5 marks]
[*5 markah*]

CLO2
C3

(b) Solve the following integrals:

Selesaikan kamiran yang berikut:

i. $\int \frac{3x}{(x+3)(x+2)}dx$

[6 marks]
[*6 markah*]

ii. $\int_0^1 \frac{1}{1+9x^2}dx$

[6 marks]
[*6 markah*]

SECTION C
BAHAGIAN C

INSTRUCTION:
This section consists of **TWO (2)** questions with 25 marks each. Answer **ONE (1)** question from each part, and **ONE (1)** remaining question from either part A/B/C

ARAHAN :
*Bahagian ini mengandungi **DUA (2)** soalan dengan jumlah 25 markah setiap soalan. Jawab SATU (1) soalan dari setiap bahagian, dan SATU (1) soalan selebihnya dari mana-mana bahagian samada A/B/C.*

QUESTION 5
SOALAN 5

CLO3
C3

(a) Form a differential equation for each of the following functions:

Bentukkan persamaan pembezaan bagi setiap fungsi yang berikut:

i. $y = Ax^3 - 6Bx$ [6 marks]
[*6 markah*]

ii. $y = 4A\cosh 2x - 4B\sinh 2x$ [4 marks]
[4 *markah*]

CLO3
C4

(b) Solve the following differential equations:

Selesaikan persamaan pembezaan berikut:

i. $e^{y-x}\frac{dy}{dx} = e^x$ [3 marks]
[*3 markah*]

ii. $\frac{dy}{dx} = \frac{xy}{x^2-y^2}$ [5 marks]
[*5 markah*]

iii. $x^2\frac{dy}{dx} + xy = 3x^3$ [7 marks]
[*7 markah*]

QUESTION 6
SOALAN 6

CLO3
C3

(a) Solve the following differential equations:
Selesaikan persamaan pembezaan berikut:

i. $$\frac{dy}{dx} = \frac{5x^2}{3y^2 + 7}$$

[6 marks]
[*6 markah*]

ii. $$\frac{dy}{dx} + 5y = e^{5x}$$

[5 marks]
[*5 markah*]

CLO3
C3

(b) Solve the following second order of differential equations:
Selesaikan persamaan pembezaan peringkat kedua berikut:

i. $$\frac{d^2y}{dx^2} + 9\frac{dy}{dx} + 20y = 0$$

[4 marks]
[*4 markah*]

ii. $$\frac{d^2y}{dx^2} + 14\frac{dy}{dx} + 49y = 0$$

[4 marks]
[*4 markah*]

iii. $$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 7y = 0$$

[6 marks]
[*6 markah*]

SOALAN TAMAT

HYPERBOLIC FUNCTIONS

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$coth\, x = \frac{e^x + e^{-x}}{e^x - e^{-x}}; x \neq 0$$

$$\sec h\, x = \frac{2}{e^x + e^{-x}}$$

$$cosech\, x = \frac{2}{e^x - e^{-x}}; x \neq 0$$

INVERSE HYPERBOLIC FUNCTIONS

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right); -\infty < x < \infty$$

$$\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right); x \geq 1$$

$$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right); |x| < 1$$

$$\coth^{-1} x = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right); |x| > 1$$

$$\text{sech}^{-1} x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right); 0 < x \leq 1$$

$$\text{cosech}^{-1} = \ln\left(\frac{1}{x} + \frac{\sqrt{1+x^2}}{|x|}\right); x \neq 0$$

RECIPROCAL TRIGONOMETRIC IDENTITIES

$$\cos ec\, x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

RECIPROCAL HYPERBOLIC IDENTITIES

$$\cos ech\, x = \frac{1}{\sinh x}$$

$$\sec h\, x = \frac{1}{\cosh x}$$

$$\coth x = \frac{1}{\tanh x}$$

TRIGONOMETRIC IDENTITIES

$$\cos^2 x + \sin^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\cot^2 x + 1 = \text{cosec}^2 x$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 2\cos^2 x - 1$$
$$= 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

HYPERBOLIC IDENTITIES

$$\cosh^2 x - \sinh^2 x = 1$$

$$1 - \tanh^2 x = \text{sech}^2 x$$

$$\coth^2 x - 1 = \text{cosech}^2 x$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$
$$= 2\cosh^2 x - 1$$
$$= 1 + 2\sinh^2 x$$

$$\tanh 2x = \frac{2\tanh x}{1 + \tanh^2 x}$$

$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$$

$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

$$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$$

BASIC OF DIFFERENTIATION	BASIC OF INTEGRATION
$\frac{d}{dx}(k) = 0;\ k = \text{constant}$	$\int k\,du = ku + C\ ;\ k = \text{constant}$
$\frac{d}{dx}(u^n) = nu^{n-1}$	$\int u^n\,du = \frac{u^{n+1}}{n+1} + C\ ;\ n \neq -1$
$\frac{d}{dx}(\ln\lvert u\rvert) = \frac{1}{u}\cdot\frac{du}{dx}$	$\int \frac{1}{u}\,du = \frac{\ln\lvert u\rvert}{u'} + C$
$\frac{d}{dx}(e^u) = e^u\cdot\frac{du}{dx}$	$\int e^u\,du = \frac{e^u}{u'} + C$

DIFFERENTIATION OF TRIGONOMETRIC FUNCTIONS	INTEGRATION OF TRIGONOMETRIC FUNCTIONS
$\frac{d}{dx}(\cos u) = -\sin u.\frac{du}{dx}$	$\int \sin u\,du = \frac{-\cos u}{u'} + C$
$\frac{d}{dx}(\sin u) = \cos u.\frac{du}{dx}$	$\int \cos u\,du = \frac{\sin u}{u'} + C$
$\frac{d}{dx}(\tan u) = \sec^2 u.\frac{du}{dx}$	$\int \sec^2 u\,du = \frac{\tan u}{u'} + C$
$\frac{d}{dx}(\cot u) = -\operatorname{cosec}^2 u.\frac{du}{dx}$	$\int \operatorname{cosec}^2 u\,du = \frac{-\cot u}{u'} + C$
$\frac{d}{dx}(\sec u) = \sec u.\tan u.\frac{du}{dx}$	$\int \sec u\tan u\,du = \frac{\sec u}{u'} + C$
$\frac{d}{dx}(\operatorname{cosec} u) = -\operatorname{cosec} u.\cot u.\frac{du}{dx}$	$\int \operatorname{cosec} u\cot u\,du = \frac{-\operatorname{cosec} u}{u'} + C$

DIFFERENTIATION OF HYPERBOLIC FUNCTIONS	INTEGRATION OF HYPERBOLIC FUNCTIONS
$\frac{d}{dx}(\cosh u) = \sinh u.\frac{du}{dx}$	$\int \sinh u\,du = \frac{\cosh u}{u'} + C$
$\frac{d}{dx}(\sinh u) = \cosh u.\frac{du}{dx}$	$\int \cosh u\,du = \frac{\sinh u}{u'} + C$
$\frac{d}{dx}(\tanh u) = \operatorname{sec} h^2 u.\frac{du}{dx}$	$\int \operatorname{sec} h^2 u\,du = \frac{\tanh u}{u'} + C$
$\frac{d}{dx}(\coth u) = -\operatorname{cosec} ech^2 u.\frac{du}{dx}$	$\int \operatorname{cosec} ech^2 u\,du = \frac{-\coth u}{u'} + C$
$\frac{d}{dx}(\operatorname{sec} h\,u) = -\operatorname{sec} h\,u.\tanh u.\frac{du}{dx}$	$\int \operatorname{sec} h\,u\tanh u\,du = \frac{-\operatorname{sec} h\ u}{u'} + C$
$\frac{d}{dx}(\operatorname{cosec} ech\ u) = -\operatorname{cosec} ech\,u.\coth u.\frac{du}{dx}$	$\int \operatorname{cosec} ech\,u\coth u\,du = \frac{-\operatorname{cosec} ech\ u}{u'} + C$

DIFFERENTIATION OF INVERSE TRYGONOMETRIC FUNCTIONS	INTEGRATION OF INVERSE TRYGONOMETRIC FUNCTION
$\frac{d}{dx}\left(\sin^{-1} u\right) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}, \lvert u\rvert < 1$	$\int \frac{1}{\sqrt{a^2-u^2}}du = \sin^{-1}\frac{u}{a} + C, \lvert u\rvert < a$
$\frac{d}{dx}\left(\cos^{-1} u\right) = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}, \lvert u\rvert < 1$	$\int -\frac{1}{\sqrt{a^2-u^2}}du = \cos^{-1}\frac{u}{a} + C, \lvert u\rvert < a$
$\frac{d}{dx}\left(\tan^{-1} u\right) = \frac{1}{1+u^2}\frac{du}{dx}$	$\int \frac{1}{a^2+u^2}du = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$
$\frac{d}{dx}\left(\cot^{-1} u\right) = -\frac{1}{1+u^2}\frac{du}{dx}$	$\int -\frac{1}{a^2+u^2}du = \frac{1}{a}\cot^{-1}\frac{u}{a} + C$
$\frac{d}{dx}\left(\sec^{-1} u\right) = \frac{1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx}, \lvert u\rvert > 1$	$\int \frac{1}{\lvert u\rvert\sqrt{u^2-a^2}}du = \frac{1}{a}\sec^{-1}\frac{u}{a} + C, \lvert u\rvert > a$
$\frac{d}{dx}\left(\cos ec^{-1} u\right) = -\frac{1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx}, \lvert u\rvert > 1$	$\int -\frac{1}{\lvert u\rvert\sqrt{u^2-a^2}}du = \frac{1}{a}\text{cosec}^{-1}\frac{u}{a} + C, \lvert u\rvert > a$

DIFFERENTIATION OF INVERSE HYPERBOLIC FUNCTIONS	INTEGRATION OF INVERSE HYPERBOLIC FUNCTIONS
$\frac{d}{dx}\left(\sinh^{-1} u\right) = \frac{1}{\sqrt{u^2+1}}\frac{du}{dx}$	$\int \frac{1}{\sqrt{a^2+u^2}}du = \sinh^{-1}\frac{u}{a} + C, \quad a > 0$
$\frac{d}{dx}\left(\cosh^{-1} u\right) = \frac{1}{\sqrt{u^2-1}}\frac{du}{dx}, \lvert u\rvert > 1$	$\int \frac{1}{\sqrt{u^2-a^2}}du = \cosh^{-1}\frac{u}{a} + C, \quad u > a$
$\frac{d}{dx}\left(\tanh^{-1} u\right) = \frac{1}{1-u^2}\frac{du}{dx}, \lvert u\rvert < 1$	$\int \frac{1}{a^2-u^2}du = \frac{1}{a}\tanh^{-1}\frac{u}{a} + C; \lvert u\rvert < a$
$\frac{d}{dx}\left(\coth^{-1} u\right) = \frac{1}{1-u^2}\frac{du}{dx}, \lvert u\rvert > 1$	$\int \frac{1}{u^2-a^2}du = \frac{1}{a}\coth^{-1}\frac{u}{a} + C; \lvert u\rvert > a$
$\frac{d}{dx}\left(\sec h^{-1} u\right) = -\frac{1}{u\sqrt{1-u^2}}\frac{du}{dx}, 0 < u < 1$	$\int \frac{1}{u\sqrt{a^2-u^2}}du = -\frac{1}{a}\text{sech}^{-1}\frac{u}{a} + C$
$\frac{d}{dx}\left(\cos ech^{-1} u\right) = -\frac{1}{\lvert u\rvert\sqrt{1+u^2}}\frac{du}{dx}, \quad u \neq 0$	$\int \frac{1}{u\sqrt{a^2+u^2}}du = -\frac{1}{a}\text{cosech}^{-1}\frac{u}{a} + C$

INTERGRALS INVOLVING QUADRATIC EXPRESSION

Completing the square

$$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

SOLUTION FOR 1st ORDER DIFFERENTIAL EQUATION

Homogeneous Equations

- Substitution

$$y = vx \quad \text{and} \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Linear Factors (Integrating Factors)

$$y \bullet IF = \int Q \bullet IF\, dx$$

Where $IF = e^{\int P\, dx}$

Logarithmic

$$a = e^{\ln a}$$

$$a^x = e^{x \ln a}$$

GENERAL SOLUTION FOR 2nd ORDER DIFFERENTIAL EQUATION

Equation of the form $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$

1. Real & different roots: $y = Ae^{m_1 x} + Be^{m_2 x}$
2. Real & equal roots: $y = e^{mx}(A + Bx)$
3. Complex roots: $y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

ROOTS OF QUADRATIC EQUATIONS

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$