

SULIT



BAHAGIAN PEPERIKSAAN DAN PENILAIAN
JABATAN PENDIDIKAN POLITEKNIK
KEMENTERIAN PENDIDIKAN TINGGI

JABATAN MATEMATIK, SAINS DAN KOMPUTER

PEPERIKSAAN AKHIR
SESI JUN 2015

BA601: ENGINEERING MATHEMATICS 5

TARIKH : 26 OKTOBER 2015
MASA : 2.30 PM - 4.30 PM (2 JAM)

Kertas ini mengandungi **EMPAT BELAS (14)** halaman bercetak.

Bahagian A: Struktur (2 soalan)

Bahagian B: Struktur (2 soalan)

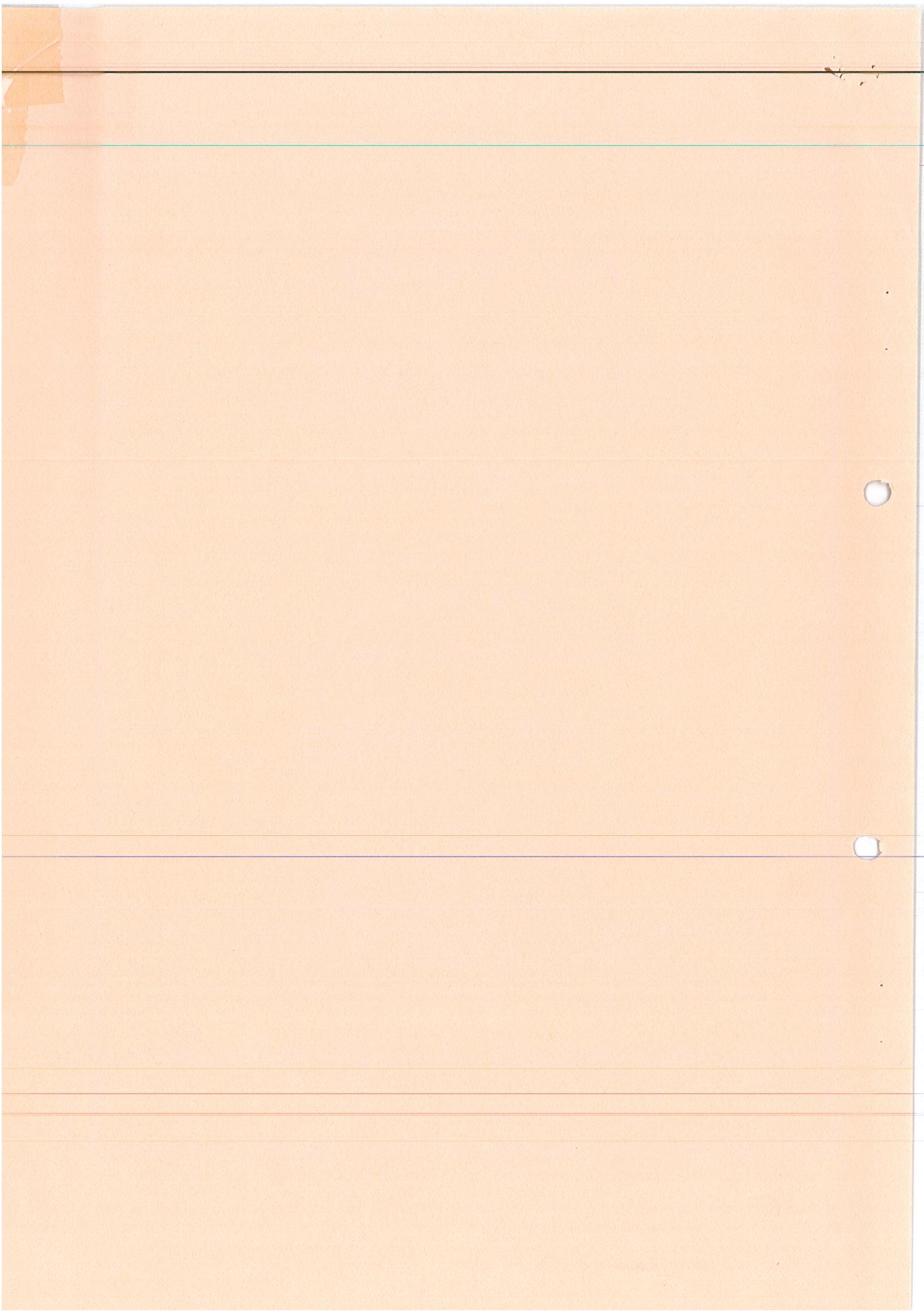
Bahagian C: Struktur (2 soalan)

Jawab 1 soalan dari setiap Bahagian dan pilih 1 soalan dari mana-mana Bahagian
Dokumen sokongan yang disertakan : Formula

JANGAN BUKA KERTAS SOALANINI SEHINGGA DIARAHKAN

(CLO yang tertera hanya sebagai rujukan)

SULIT



INSTRUCTION:

Answer **ONE (1)** question from each section (A, B and C) and answer **ONE (1)** question from any section that has not been answered.

ARAHAN :

Jawab SATU (1) soalan daripada setiap bahagian (A, B dan C) dan Jawab SATU (1) lagi soalan yang belum dijawab dari mana-mana bahagian.

SECTION A
BAHAGIAN A

QUESTION 1
SOALAN 1

CLO1
C2

- (a) By using definition of hyperbolic functions, find the value of:

Dengan menggunakan definisi fungsi hiperbola, cari nilai bagi:

i. $\sinh(-3.254)$

[2 marks]
[2 markah]

ii. $4 \operatorname{sech}(\sqrt{8})$

[2 marks]
[2 markah]

iii. $\frac{8 \coth(2.48)}{\operatorname{cosech}\left(\frac{3}{4}\right)}$

[3 marks]
[3 markah]

CLO1
C2

- (b) If $\sinh \frac{3x}{5} + k^3 = 2z^2 \coth(x + z^2)$, find the value of k when $x = 2$ and $z = 4$.

Jika $\sinh \frac{3x}{5} + k^3 = 2z^2 \coth(x + z^2)$, cari nilai bagi k bila $x = 2$ dan $z = 4$.

[5 marks]
[5 markah]

- CLO1 (c) Prove that $\coth^2 x - 1 = \operatorname{cosech}^2 x$ by using definition of hyperbolic functions.
C2

Buktikan bahawa $\coth^2 x - 1 = \operatorname{cosech}^2 x$ dengan menggunakan definisi fungsi hiperbolik.

[7 marks]
[7 markah]

- CLO1 (d) Complete the table below for equation $y = \frac{\cosh x}{3}$. Then sketch the graph in the
C2 range of $-3 \leq x \leq 3$.

Lengkapkan jadual di bawah bagi persamaan $y = \frac{\cosh x}{3}$. Seterusnya, lakarkan graf pada julat $-3 \leq x \leq 3$.

[6 marks]
[6 markah]

x	-3	-2	-1	0	1	2	3
y							

QUESTION 2**SOALAN 2**CLO1
C2

- (a) Find the value of the following functions by using the definition of hyperbolic functions.

Cari nilai bagi fungsi-fungsi yang berikut dengan menggunakan definisi fungsi hiperbolik.

i. $\sinh(\ln 2.7)$

[2 marks]
[2 markah]

ii. $\operatorname{cosech}\sqrt{2.5}$

[2 marks]
[2 markah]

iii. $\frac{\sinh 2.5}{\cosh(-0.54)}$

[3 marks]
[3 markah]

CLO1
C2

- (b) Find the value for $\sinh x$ if $2 \cosh x = 3$.

Cari nilai bagi $\sinh x$ jika $2 \cosh x = 3$.

[5 marks]
[5 markah]

CLO1
C2

- (c) Complete the table below for the equation of $y = \sec h\left(\frac{x}{2}\right)$ and sketch the graph.

Lengkapkan jadual di bawah bagi persamaan $y = \sec h\left(\frac{x}{2}\right)$ dan lakarkan graf.

x	-4	-2	0	2	4
y					

[5 marks]
[5 markah]

CLO1
C3

- (d) Sketch a quadrant graph and determine the principal value for each of the following functions given:

Lakarkan graf sukuan dan dapatkan nilai prinsip bagi setiap fungsi yang diberikan:

i. $\cos^{-1}(0.8557)$

[4 marks]
[4 markah]

ii. $\tan^{-1}(-1.75)$

[4 marks]
[4 markah]

SECTION B
BAHAGIAN B

QUESTION 3
SOALAN 3

- CLO2
C3 (a) Differentiate the following functions in respect to x .
Bezakan setiap fungsi yang berikut terhadap x .

i. $y = \cot^{-1}(e^x)$

[3 marks]
[3 markah]

ii. $y = \cosech^{-1}(2x)$

[3 marks]
[3 markah]

iii. $y = x^2 \ln(\sinh x)$

[5 marks]
[5 markah]

iv. $y = \cosh^{-1} \sec(x)$

[5 marks]
[5 markah]

v. $y = (\sinh x)(\coth x)$

[3 marks]
[3 markah]

CLO2
C3

(b) Prove that $\frac{d}{dx}(\cosh^{-1}(\cosh x)) = 1$

Buktikan bahawa $\frac{d}{dx}(\cosh^{-1}(\cosh x)) = 1$

[6 marks]
[6 markah]

QUESTION 4**SOALAN 4**CLO2
C3

(a) Determine the following integrals :

Tentukan setiap kamiran yang berikut:

i. $\int \frac{2x}{x^2 + 4} dx$

[4 marks]
[4 markah]

ii. $\int \frac{1}{(x+1)\sqrt{x^2 + 2x}} dx$

[6 marks]
[6 markah]

iii. $\int \sinh(2x) dx$

[3 marks]
[3 markah]CLO2
C3

(b) Solve the following integrals given :

Selesaikan kamiran berikut:

i. $\int x^2 \ln(3x) dx$

[6 marks]
[6 markah]

ii. $\int \frac{x-1}{2x^2 - x - 3} dx$

[6 marks]
[6 markah]

SECTION C
BAHAGIAN C

QUESTION 5
SOALAN 5

CLO3

C3

(a) Form a differential equation for each of the following functions:

Bentukkan persamaan pembezaan bagi setiap fungsi yang berikut :

i. $y = Ax^3 + x^4$

[5 marks]
[5 markah]

ii. $y = A\cos(3x + B)$

[3 marks]
[3 markah]

CLO3

C3

(b) Solve the following differential equations:

Selesaikan persamaan pembezaan berikut:

i. $y \sqrt{(1+x^2)} dy = -x \sqrt{(1+y^2)} dx$

[7 marks]
[7 markah]

ii. $x \frac{dy}{dx} - 5y = x^7$

[6marks]
[6 markah]

CLO3

C3

(c) Solve the second order differential equation below.

Selesaikan persamaan pembezaan peringkat kedua di bawah.

$$4 \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$$

[4 marks]
[4 markah]

QUESTION 6
SOALAN 6

CLO3
C3

- (a) Form a differential equation for $y = Ax + \frac{B}{x}$.

Bentukkan persamaan pembezaan bagi $y = Ax + \frac{B}{x}$.

[7 marks]
[7 markah]

CLO3
C3

- (b) Solve the differential equation for $\frac{dy}{dx} = \frac{2y^3 - x^3}{3xy^2}$.

Selesaikan persamaan pembezaan bagi $\frac{dy}{dx} = \frac{2y^3 - x^3}{3xy^2}$.

[8 marks]
[8 markah]

CLO3
C3

- (c) Solve the following second order of differential equations:

Selesaikan persamaan pembezaan peringkat kedua berikut:

i. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

[3 marks]
[3 markah]

ii. $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 41y = 0$

[7 marks]
[7 markah]

SOALAN TAMAT

HYPERBOLIC FUNCTIONS	INVERSE HYPERBOLIC FUNCTIONS
$\sinh x = \frac{e^x - e^{-x}}{2}$ $\cosh x = \frac{e^x + e^{-x}}{2}$ $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ $\coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}}; x \neq 0$ $\sec h x = \frac{2}{e^x + e^{-x}}$ $\operatorname{cosech} x = \frac{2}{e^x - e^{-x}}; x \neq 0$	$\sinh^{-1} x = \ln \left(x + \sqrt{x^2 + 1} \right); -\infty < x < \infty$ $\cosh^{-1} x = \ln \left(x + \sqrt{x^2 - 1} \right); x \geq 1$ $\tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right); x < 1$ $\coth^{-1} x = \frac{1}{2} \ln \left(\frac{x+1}{x-1} \right); x > 1$ $\operatorname{sech}^{-1} x = \ln \left(\frac{1 + \sqrt{1 - x^2}}{x} \right); 0 < x \leq 1$ $\operatorname{cosech}^{-1} x = \ln \left(\frac{1}{x} + \frac{\sqrt{1 + x^2}}{ x } \right); x \neq 0$
RECIPROCAL TRIGONOMETRIC IDENTITIES	RECIPROCAL HYPERBOLIC IDENTITIES
$\operatorname{cosec} x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$	$\operatorname{cosech} x = \frac{1}{\sinh x}$ $\sec h x = \frac{1}{\cosh x}$ $\coth x = \frac{1}{\tanh x}$
TRIGONOMETRIC IDENTITIES	HYPERBOLIC IDENTITIES
$\cos^2 x + \sin^2 x = 1$ $1 + \tan^2 x = \sec^2 x$ $\cot^2 x + 1 = \operatorname{cosec}^2 x$	$\cosh^2 x - \sinh^2 x = 1$ $1 - \tanh^2 x = \operatorname{sech}^2 x$ $\coth^2 x - 1 = \operatorname{cosech}^2 x$

$\sin 2x = 2 \sin x \cos x$	$\sinh 2x = 2 \sinh x \cosh x$
$\cos 2x = \cos^2 x - \sin^2 x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$= 2 \cos^2 x - 1$	$= 2 \cosh^2 x - 1$
$= 1 - 2 \sin^2 x$	$= 1 + 2 \sinh^2 x$
$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$	$\tanh 2x = \frac{2 \tanh x}{1 + \tanh^2 x}$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$	$\sinh(x \pm y) =$ $\sinh x \cosh y \pm \cosh x \sinh y$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$
$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$

BASIC OF DIFFERENTIATION	BASIC OF INTEGRATION
$\frac{d}{dx}(k) = 0; k = \text{constant}$	$\int k \, du = ku + C; k = \text{constant}$
$\frac{d}{dx}(u^n) = nu^{n-1}$	$\int u^n \, du = \frac{u^{n+1}}{n+1} + C; n \neq -1$
$\frac{d}{dx}(\ln u) = \frac{1}{u} \cdot \frac{du}{dx}$	$\int \frac{1}{u} \, du = \frac{\ln u }{u'} + C$
$\frac{d}{dx}(e^u) = e^u \cdot \frac{du}{dx}$	$\int e^u \, du = \frac{e^u}{u'} + C$

DIFFERENTIATION OF TRIGONOMETRIC FUNCTIONS	INTEGRATION OF TRIGONOMETRIC FUNCTIONS
$\frac{d}{dx}(\cos u) = -\sin u \cdot \frac{du}{dx}$	$\int \sin u \, du = \frac{-\cos u}{u'} + C$
$\frac{d}{dx}(\sin u) = \cos u \cdot \frac{du}{dx}$	$\int \cos u \, du = \frac{\sin u}{u'} + C$
$\frac{d}{dx}(\tan u) = \sec^2 u \cdot \frac{du}{dx}$	$\int \sec^2 u \, du = \frac{\tan u}{u'} + C$
$\frac{d}{dx}(\cot u) = -\operatorname{cosec}^2 u \cdot \frac{du}{dx}$	$\int \operatorname{cosec}^2 u \, du = \frac{-\cot u}{u'} + C$

$\frac{d}{dx}(\sec u) = \sec u \cdot \tan u \cdot \frac{du}{dx}$	$\int \sec u \tan u du = \frac{\sec u}{u'} + C$
$\frac{d}{dx}(\csc u) = -\csc u \cdot \cot u \cdot \frac{du}{dx}$	$\int \csc u \cot u du = \frac{-\csc u}{u'} + C$
DIFFERENTIATION OF HYPERBOLIC FUNCTIONS	INTEGRATION OF HYPERBOLIC FUNCTIONS
$\frac{d}{dx}(\cosh u) = \sinh u \cdot \frac{du}{dx}$	$\int \sinh u du = \frac{\cosh u}{u'} + C$
$\frac{d}{dx}(\sinh u) = \cosh u \cdot \frac{du}{dx}$	$\int \cosh u du = \frac{\sinh u}{u'} + C$
$\frac{d}{dx}(\tanh u) = \operatorname{sech}^2 u \cdot \frac{du}{dx}$	$\int \operatorname{sech}^2 u du = \frac{\tanh u}{u'} + C$
$\frac{d}{dx}(\coth u) = -\operatorname{cosech}^2 u \cdot \frac{du}{dx}$	$\int \operatorname{cosech}^2 u du = \frac{-\coth u}{u'} + C$
$\frac{d}{dx}(\operatorname{sech} u) = -\operatorname{sech} u \cdot \tanh u \cdot \frac{du}{dx}$	$\int \operatorname{sech} u \tanh u du = \frac{-\operatorname{sech} u}{u'} + C$
$\frac{d}{dx}(\operatorname{cosech} u) = -\operatorname{cosech} u \cdot \coth u \cdot \frac{du}{dx}$	$\int \operatorname{cosech} u \coth u du = \frac{-\operatorname{cosech} u}{u'} + C$
DIFFERENTIATION OF INVERSE TRYGONOMETRIC FUNCTIONS	INTEGRATION OF INVERSE TRYGONOMETRIC FUNCTION
$\frac{d}{dx}(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}, u < 1$	$\int \frac{1}{\sqrt{a^2-u^2}} du = \sin^{-1} \frac{u}{a} + C, u < a$
$\frac{d}{dx}(\cos^{-1} u) = -\frac{1}{\sqrt{1-u^2}} \frac{du}{dx}, u < 1$	$\int -\frac{1}{\sqrt{a^2-u^2}} du = \cos^{-1} \frac{u}{a} + C, u < a$
$\frac{d}{dx}(\tan^{-1} u) = \frac{1}{1+u^2} \frac{du}{dx}$	$\int \frac{1}{a^2+u^2} du = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$
$\frac{d}{dx}(\cot^{-1} u) = -\frac{1}{1+u^2} \frac{du}{dx}$	$\int -\frac{1}{a^2+u^2} du = \frac{1}{a} \cot^{-1} \frac{u}{a} + C$
$\frac{d}{dx}(\sec^{-1} u) = \frac{1}{ u \sqrt{u^2-1}} \frac{du}{dx}, u > 1$	$\int \frac{1}{ u \sqrt{u^2-a^2}} du = \frac{1}{a} \sec^{-1} \frac{u}{a} + C, u > a$

$$\frac{d}{dx} (\cos ec^{-1} u) = -\frac{1}{|u|\sqrt{u^2-1}} \frac{du}{dx}, |u| > 1$$

$$\int -\frac{1}{|u|\sqrt{u^2-a^2}} du = \frac{1}{a} \operatorname{cosec}^{-1} \frac{u}{a} + C, |u| > a$$

DIFFERENTIATION OF INVERSE HYPERBOLIC FUNCTIONS

$$\frac{d}{dx} (\sinh^{-1} u) = \frac{1}{\sqrt{u^2+1}} \frac{du}{dx}$$

$$\frac{d}{dx} (\cosh^{-1} u) = \frac{1}{\sqrt{u^2-1}} \frac{du}{dx}, |u| > 1$$

$$\frac{d}{dx} (\tanh^{-1} u) = \frac{1}{1-u^2} \frac{du}{dx}, |u| < 1$$

$$\frac{d}{dx} (\coth^{-1} u) = \frac{1}{1-u^2} \frac{du}{dx}, |u| > 1$$

$$\frac{d}{dx} (\sec h^{-1} u) = -\frac{1}{u\sqrt{1-u^2}} \frac{du}{dx}, 0 < u < 1$$

$$\frac{d}{dx} (\cos ech^{-1} u) = -\frac{1}{|u|\sqrt{1+u^2}} \frac{du}{dx}, u \neq 0$$

INTEGRATION OF INVERSE HYPERBOLIC FUNCTIONS

$$\int \frac{1}{\sqrt{a^2+u^2}} du = \sinh^{-1} \frac{u}{a} + C, a > 0$$

$$\int \frac{1}{\sqrt{u^2-a^2}} du = \cosh^{-1} \frac{u}{a} + C, u > a$$

$$\int \frac{1}{a^2-u^2} du = \frac{1}{a} \tanh^{-1} \frac{u}{a} + C; |u| < a$$

$$\int \frac{1}{u^2-a^2} du = \frac{1}{a} \coth^{-1} \frac{u}{a} + C; |u| > a$$

$$\int \frac{1}{u\sqrt{a^2-u^2}} du = -\frac{1}{a} \operatorname{sech}^{-1} \frac{u}{a} + C$$

$$\int \frac{1}{u\sqrt{a^2+u^2}} du = -\frac{1}{a} \operatorname{cosech}^{-1} \frac{u}{a} + C$$

INTERGRALS INVOLVING QUADRATIC EXPRESSION

Completing the square

$$ax^2 + bx + c = a \left(x + \frac{b}{2a} \right)^2 + c - \frac{b^2}{4a}$$

SOLUTION FOR 1st ORDER DIFFERENTIAL EQUATION**Homogeneous Equations**

- Substitution

$$y = vx \quad \text{and} \quad \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Linear Factors (Integrating Factors)

$$y \bullet IF = \int Q \bullet IF dx$$

Where $IF = e^{\int P dx}$

Logarithmic

$$a = e^{\ln a}$$

$$a^x = e^{x \ln a}$$

GENERAL SOLUTION FOR 2nd ORDER DIFFERENTIAL EQUATION

Equation of the form $a \frac{d^2 y}{dx^2} + b \frac{dy}{dx} + cy = 0$

$$y = Ae^{m_1 x} + Be^{m_2 x}$$

1. Real & different roots:

$$y = e^{mx}(A + Bx)$$

2. Real & equal roots:

$$y = e^{\alpha x}(A \cos \beta x + B \sin \beta x)$$

3. Complex roots:

ROOTS OF QUADRATIC EQUATIONS

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

