9

SULIT

BAHAGIAN PEPERIKSAAN DAN PENILAIAN
JABATAN PENDIDIKAN POLITEKNIK
KEMENTERIAN PENDIDIKAN TINGGI

JABATAN MATEMATIK, SAINS & KOMPUTER

PEPERIKSAAN AKHIR

SESI JUN 2016

BA601: ENGINEERING MATHEMATICS 5

TARIKH : 27 OKTOBER 2016
MASA : 8.30 AM - 10.30 AM (2 JAM)

Kertas ini mengandungi **SEMBILAN (9)** halaman bercetak.
Bahagian A: Struktur (2 soalan)
Bahagian B: Struktur (2 soalan)
Bahagian C: Struktur (2 soalan)
Dokumen sokongan yang disertakan : Formula

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIARAHKAN

(CLO yang tertera hanya sebagai rujukan)

SULIT

INSTRUCTION:

Answer **ONE (1)** question from each section **(A, B and C)** and answer **ONE (1)** question from any section that has not been answered.

ARAHAN :
*Jawab **SATU (1)** soalan daripada setiap bahagian **(A,B dan C)** dan jawab **SATU (1)** soalan yang belum dijawab dari mana-mana bahagian.*

SECTION A
BAHAGIAN A

QUESTION 1
SOALAN 1

CLO 1
C1

a) Find the value for each of the following by using the definition of hyperbolic functions :
Cari nilai bagi setiap yang berikut dengan menggunakan definisi fungsi hiperbolik :

i) $\cosh(\ln 3)$

[2 marks]
[*2 markah*]

ii) $\tanh\sqrt{4}$

[2 marks]
[*2 markah*]

iii) $\sinh x = \frac{5}{12}$, find $\tanh x$

[5 marks]
[*5 markah*]

iv) $cosech\ \frac{5}{12}$

[3 marks]
[*3 markah*]

CLO 1
C2

b) If the water wave with length, $L = 40$ m moves with velocity in the depth of water, d = 5 m where g is the gravity acceleration = 9.81 ms^{-2}. Find the velocity, v.

Jika panjang ombak L = 40 m bergerak dengan kelajuan kedalaman air, d = 5 m di mana g adalah pecutan graviti iaitu = 9.81 ms^{-2}. Cari nilai kelajuan, v.

$$v = \sqrt{\frac{gL}{2\pi}\tanh(\frac{2\pi d}{L})}$$

[5 marks]

[5 markah]

CLO 1
C3

c) Prove that $\sinh(x + y) = sinh\,x\,cosh\,y + cosh\,x\,sinh\,y$.

Buktikan bahawa $\sinh(x + y) = sinh\,x\,cosh\,y + cosh\,x\,sinh\,y$.

[5 marks]

[5 markah]

CLO 1
C3

d) Determine the principle value of $cos^{-1}(\cos 50°)$. Sketch the graph.

Dapatkan nilai prinsipal bagi $cos^{-1}(\cos 50°)$. *Lakarkan graf.*

[3 marks]

[3 markah]

QUESTION 2
SOAALAN 2

CLO1
C2

(a) Calculate the value for each of the following hyperbolic functions.

Kirakan nilai bagi setiap fungsi hyperbola yang berikut.

i. $\sinh(-5)$

[2 marks]
[*2 markah*]

ii. $\tanh\left(\frac{1}{3}\right)$

[2 marks]
[*2 markah*]

iii. $\coth^{-1} 5$

[2 marks]
[*2 markah*]

iv. $\text{sech}^{-1}(0.2)$

[2 marks]
[*2 markah*]

CLO1
C3

(b) Prove that :
Buktikan :

$$\cosh x + \sinh x = e^x$$

[4 marks]
[*4 markah*]

CLO1
C2

(c) Sketch a quadrant graph and find the principal value for the following functions.
Lakarkan graf sukuan dan dapatkan nilai prinsipal bagi fungsi-fungsi berikut.

i. $\sin^{-1}(0.95)$

[4 marks]
[*4 markah*]

ii. $\tan^{-1}(\sqrt{2})$

[4 marks]
[*4 markah*]

iii. $\cot^{-1}(\sqrt{9})$

[5 marks]
[*5 markah*]

SECTION B
BAHAGIAN B

QUESTION 3
SOALAN 3

CLO2 C2

a) Find $\frac{dy}{dx}$ for the following functions:

Cari $\frac{dy}{dx}$ *bagi fungsi berikut:*

i) $y = -2\sinh^3 3x$ [3 marks]

[*3 markah*]

ii) $y = \cosh^{-1}(\tan x)$ [4 marks]

[*4 markah*]

iii) $y = e^{2x}\sinh^{-1} 5x$ [5 marks]

[*5 markah*]

CLO2 C2

b) Differentiate the following function to x

Bezakan fungsi berikut terhadap x

$$5x^3 + 4x^2 y + y^3 = \cosh x$$

[6 marks]
[*6 markah*]

CLO2 C3

c) If $z = 3x\cos y$, prove that $\frac{\partial^2 z}{\partial y \partial x} + \frac{\partial^2 z}{\partial x \partial y} = -6\sin y$

Jika $z = 3x\cos y$ *buktikan* $\frac{\partial^2 z}{\partial y \partial x} + \frac{\partial^2 z}{\partial x \partial y} = -6\sin y$

[7 marks]
[*7 markah*]

QUESTION 4
SOALAN 4

CLO2
C3

(a) Find the integration of the following functions.
Dapatkan pengamiran fungsi dibawah.

i. $\int 7\tanh(3x+\frac{1}{2})dx$

[5 marks]
[*5 markah*]

ii. $\int \frac{1}{\sqrt{25-4x^2}}dx$

[5 marks]
[*5 markah*]

iii. $\int \frac{1}{\sqrt{9x^2-3}}dx$

[5 marks]
[*5 markah*]

iv. $\int \frac{e^{2x}}{1+e^{4x}}dx$

[5 marks]
[*5 markah*]

CLO2
C3

(b) Integrate $\frac{x}{(x-3)(x+3)}$ to x by using **integration by partial fractions.**

Kamirkan $\frac{x}{(x-3)(x+3)}$ *terhadap x dengan menggunakan* ***kamiran pecahan separa.***

[5 marks]
[*5 markah*]

SECTION C
BAHAGIAN C

QUESTION 5
SOALAN 5

CLO3 C2 (a) Form the differential equation for $y = Ax^2 - Bx + x$.

Bentukkan persamaan pembezaan bagi $y = Ax^2 - Bx + x$

[8 marks]

[*8 markah*]

CLO3 C3 (b) Determine the general solution for the following differential equations.

Tentukan penyelesaian am bagi persamaan pembezaan yang berikut.

i. $e^x \frac{dy}{dx} = 4$, given $y = 3$ and $x = 0$. [5 marks]

[5 *markah*]

ii. $x\frac{dy}{dx} = 5x^3 + 4$ [5 marks]

[5 *markah*]

iii. $x\frac{dy}{dx} + y = x^3$ [7 marks]

[7 *markah*]

QUESTION 6

SOALAN 6

CLO3 C4

(a) Solve the differential equation below by using an appropriate method.

Selesaikan persamaan pembezaan berikut dengan menggunakan kaedah yang sesuai.

$$(x^2 + xy)\frac{dy}{dx} = xy - y^2$$

[10 marks]

[*10 markah*]

CLO3 C3

(b) Solve the following second order differential equation.

Selesaikan persamaan pembezaan peringkat kedua di bawah.

i. $2\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 3y = 0$

[4 marks]

[*4 markah*]

ii. $\frac{d^2y}{dx^2} + 14\frac{dy}{dx} + 49y = 0$

[4 marks]

[*4 markah*]

iii. $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 7y = 0$

[7 marks]

[*7 markah*]

SOALAN TAMAT

HYPERBOLIC FUNCTIONS	INVERSE HYPERBOLIC FUNCTIONS
$\sinh x = \dfrac{e^x - e^{-x}}{2}$	$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right); -\infty < x < \infty$
$\cosh x = \dfrac{e^x + e^{-x}}{2}$	$\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right); x \geq 1$
$\tanh x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$	$\tanh^{-1} x = \dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right); \lvert x\rvert < 1$
$\coth x = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}; x \neq 0$	$\coth^{-1} x = \dfrac{1}{2}\ln\left(\dfrac{x+1}{x-1}\right); \lvert x\rvert > 1$
$\operatorname{sech} x = \dfrac{2}{e^x + e^{-x}}$	$\operatorname{sech}^{-1} x = \ln\left(\dfrac{1+\sqrt{1-x^2}}{x}\right); 0 < x \leq 1$
$\operatorname{cosech} x = \dfrac{2}{e^x - e^{-x}}; x \neq 0$	$\operatorname{cosech}^{-1} = \ln\left(\dfrac{1}{x} + \dfrac{\sqrt{1+x^2}}{\lvert x\rvert}\right); x \neq 0$

RECIPROCAL TRIGONOMETRIC IDENTITIES	RECIPROCAL HYPERBOLIC IDENTITIES
$\operatorname{cosec} x = \dfrac{1}{\sin x}$	$\operatorname{cosech} x = \dfrac{1}{\sinh x}$
$\sec x = \dfrac{1}{\cos x}$	$\operatorname{sech} x = \dfrac{1}{\cosh x}$
$\cot x = \dfrac{1}{\tan x}$	$\coth x = \dfrac{1}{\tanh x}$

TRIGONOMETRIC IDENTITIES	HYPERBOLIC IDENTITIES
$\cos^2 x + \sin^2 x = 1$	$\cosh^2 x - \sinh^2 x = 1$
$1 + \tan^2 x = \sec^2 x$	$1 - \tanh^2 x = \operatorname{sech}^2 x$
$\cot^2 x + 1 = \operatorname{cosec}^2 x$	$\coth^2 x - 1 = \operatorname{cosech}^2 x$
$\sin 2x = 2 \sin x \cos x$	$\sinh 2x = 2 \sinh x \cosh x$
$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$	$\cosh 2x = \cosh^2 x + \sinh^2 x = 2\cosh^2 x - 1 = 1 + 2\sinh^2 x$
$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$	$\tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$	$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$
$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$

BASIC OF DIFFERENTIATION	BASIC OF INTEGRATION
$\frac{d}{dx}(k) = 0;\ k = \text{constant}$	$\int k\,du = ku + C\ ;\ k = \text{constant}$
$\frac{d}{dx}(u^n) = nu^{n-1}$	$\int u^n\,du = \frac{u^{n+1}}{n+1} + C\ ;\ n \neq -1$
$\frac{d}{dx}(\ln\|u\|) = \frac{1}{u} \cdot \frac{du}{dx}$	$\int \frac{1}{u}\,du = \frac{\ln\|u\|}{\left(\frac{du}{dx}\right)} + C$
$\frac{d}{dx}(e^u) = e^u \cdot \frac{du}{dx}$	$\int e^u\,du = \frac{e^u}{\left(\frac{du}{dx}\right)} + C$

DIFFERENTIATION OF TRIGONOMETRIC FUNCTIONS	INTEGRATION OF TRIGONOMETRIC FUNCTIONS
$\frac{d}{dx}(\cos u) = -\sin u . \frac{du}{dx}$	$\int \sin u\,du = \frac{-\cos u}{\left(\frac{du}{dx}\right)} + C$
$\frac{d}{dx}(\sin u) = \cos u . \frac{du}{dx}$	$\int \cos u\,du = \frac{\sin u}{\left(\frac{du}{dx}\right)} + C$
$\frac{d}{dx}(\tan u) = \sec^2 u . \frac{du}{dx}$	$\int \sec^2 u\,du = \frac{\tan u}{\left(\frac{du}{dx}\right)} + C$
$\frac{d}{dx}(\cot u) = -\operatorname{cosec}^2 u . \frac{du}{dx}$	$\int \operatorname{cosec}^2 u\,du = \frac{-\cot u}{\left(\frac{du}{dx}\right)} + C$
$\frac{d}{dx}(\sec u) = \sec u . \tan u . \frac{du}{dx}$	$\int \sec u \tan u\,du = \frac{\sec u}{\left(\frac{du}{dx}\right)} + C$
$\frac{d}{dx}(\operatorname{cosec} u) = -\operatorname{cosec} u . \cot u . \frac{du}{dx}$	$\int \operatorname{cosec} u \cot u\,du = \frac{-\operatorname{cosec} u}{\left(\frac{du}{dx}\right)} + C$

DIFFERENTIATION OF HYPERBOLIC FUNCTIONS	INTEGRATION OF HYPERBOLIC FUNCTIONS
$\frac{d}{dx}(\cosh u) = \sinh u . \frac{du}{dx}$	$\int \sinh u\,du = \frac{\cosh u}{\left(\frac{du}{dx}\right)} + C$
$\frac{d}{dx}(\sinh u) = \cosh u . \frac{du}{dx}$	$\int \cosh u\,du = \frac{\sinh u}{\left(\frac{du}{dx}\right)} + C$
$\frac{d}{dx}(\tanh u) = \operatorname{sech}^2 u . \frac{du}{dx}$	$\int \operatorname{sech}^2 u\,du = \frac{\tanh u}{\left(\frac{du}{dx}\right)} + C$
$\frac{d}{dx}(\coth u) = -\operatorname{cosech}^2 u . \frac{du}{dx}$	$\int \operatorname{cosech}^2 u\,du = \frac{-\coth u}{\left(\frac{du}{dx}\right)} + C$
$\frac{d}{dx}(\operatorname{sech} u) = -\operatorname{sech} u . \tanh u . \frac{du}{dx}$	$\int \operatorname{sech} u \tanh u\,du = \frac{-\operatorname{sech} u}{\left(\frac{du}{dx}\right)} + C$
$\frac{d}{dx}(\operatorname{cosech} u) = -\operatorname{cosech} u . \coth u . \frac{du}{dx}$	$\int \operatorname{cosech} u \coth u\,du = \frac{-\operatorname{cosech} u}{\left(\frac{du}{dx}\right)} + C$

DIFFERENTIATION OF INVERSE TRYGONOMETRIC FUNCTIONS	INTEGRATION OF INVERSE TRYGONOMETRIC FUNCTION
$\frac{d}{dx}\left(\sin^{-1} u\right) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$, $\lvert u\rvert<1$	$\int \frac{1}{\sqrt{a^2-u^2}}du = \sin^{-1}\frac{u}{a}+C,\ \lvert u\rvert<a$
$\frac{d}{dx}\left(\cos^{-1} u\right) = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx},\ \lvert u\rvert<1$	$\int -\frac{1}{\sqrt{a^2-u^2}}du = \cos^{-1}\frac{u}{a}+C,\ \lvert u\rvert<a$
$\frac{d}{dx}\left(\tan^{-1} u\right) = \frac{1}{1+u^2}\frac{du}{dx}$	$\int \frac{1}{a^2+u^2}du = \frac{1}{a}\tan^{-1}\frac{u}{a}+C$
$\frac{d}{dx}\left(\cot^{-1} u\right) = -\frac{1}{1+u^2}\frac{du}{dx}$	$\int -\frac{1}{a^2+u^2}du = \frac{1}{a}\cot^{-1}\frac{u}{a}+C$
$\frac{d}{dx}\left(\sec^{-1} u\right) = \frac{1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx},\ \lvert u\rvert>1$	$\int \frac{1}{\lvert u\rvert\sqrt{u^2-a^2}}du = \frac{1}{a}\sec^{-1}\frac{u}{a}+C,\ \lvert u\rvert>a$
$\frac{d}{dx}\left(\cos ec^{-1} u\right) = -\frac{1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx},\ \lvert u\rvert>1$	$\int -\frac{1}{\lvert u\rvert\sqrt{u^2-a^2}}du = \frac{1}{a}\text{cosec}^{-1}\frac{u}{a}+C,\ \lvert u\rvert>a$

DIFFERENTIATION OF INVERSE HYPERBOLIC FUNCTIONS	INTEGRATION OF INVERSE HYPERBOLIC FUNCTIONS
$\frac{d}{dx}\left(\sinh^{-1} u\right) = \frac{1}{\sqrt{u^2+1}}\frac{du}{dx}$	$\int \frac{1}{\sqrt{a^2+u^2}}du = \sinh^{-1}\frac{u}{a}+C,\quad a>0$
$\frac{d}{dx}\left(\cosh^{-1} u\right) = \frac{1}{\sqrt{u^2-1}}\frac{du}{dx},\ \lvert u\rvert>1$	$\int \frac{1}{\sqrt{u^2-a^2}}du = \cosh^{-1}\frac{u}{a}+C,\quad u>a$
$\frac{d}{dx}\left(\tanh^{-1} u\right) = \frac{1}{1-u^2}\frac{du}{dx},\ \lvert u\rvert<1$	$\int \frac{1}{a^2-u^2}du = \frac{1}{a}\tanh^{-1}\frac{u}{a}+C;\ \lvert u\rvert<a$
$\frac{d}{dx}\left(\coth^{-1} u\right) = \frac{1}{1-u^2}\frac{du}{dx},\lvert u\rvert>1$	$\int \frac{1}{u^2-a^2}du = \frac{1}{a}\coth^{-1}\frac{u}{a}+C;\ \lvert u\rvert>a$
$\frac{d}{dx}\left(\sec h^{-1} u\right) = -\frac{1}{u\sqrt{1-u^2}}\frac{du}{dx},\ 0<u<1$	$\int \frac{1}{u\sqrt{a^2-u^2}}du = -\frac{1}{a}\text{sech}^{-1}\frac{u}{a}+C$
$\frac{d}{dx}\left(\cos ech^{-1} u\right) = -\frac{1}{\lvert u\rvert\sqrt{1+u^2}}\frac{du}{dx},\quad u\neq 0$	$\int \frac{1}{u\sqrt{a^2+u^2}}du = -\frac{1}{a}\text{cosech}^{-1}\frac{u}{a}+C$

INTERGRALS INVOLVING QUADRATIC EXPRESSION

Completing the square

$$ax^2+bx+c = a\left(x+\frac{b}{2a}\right)^2+c-\frac{b^2}{4a}$$

SOLUTION FOR 1st ORDER DIFFERENTIAL EQUATION

Homogeneous Equations

- Substitution

$y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

Linear Factors (Integrating Factors)

$$y \bullet IF = \int Q \bullet IF \, dx$$

Where $IF = e^{\int P\,dx}$

Logarithmic

$a = e^{\ln a}$

$a^x = e^{x \ln a}$

$\int a^x \, dx = \frac{a^x}{\ln a} + c$

GENERAL SOLUTION FOR 2nd ORDER DIFFERENTIAL EQUATION

Equation of the form $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$

1. Real & different roots: $y = Ae^{m_1 x} + Be^{m_2 x}$
2. Real & equal roots: $y = e^{mx}(A + Bx)$
3. Complex roots: $y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$