

8

SULIT



BAHAGIAN PEPERIKSAAN DAN PENILAIAN
JABATAN PENDIDIKAN POLITEKNIK
KEMENTERIAN PENDIDIKAN TINGGI

JABATAN MATEMATIK SAINS DAN KOMPUTER

PEPERIKSAAN AKHIR
SESI DISEMBER 2015

BA501: ENGINEERING MATHEMATICS 4

TARIKH : 05 APRIL 2016
MASA : 8.30 AM - 10.30 AM (2 JAM)

Kertas ini mengandungi **LAPAN BELAS (18)** halaman bercetak.

Bahagian A : Struktur (2 soalan)

Bahagian B : Struktur (2 soalan)

Bahagian C : Struktur (2 soalan) (JKE, JKP, JKPK)

Bahagian D : Struktur (2 soalan) (JKM, JPP)

Bahagian E : Struktur (1 soalan)

Dokumen sokongan yang disertakan : Formula

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIARAHKAN

(CLO yang tertera hanya sebagai rujukan)

SULIT

SULIT

SECTION A (JKE, JKP, JKPK, JPP and JKM): 25 MARKS

BAHAGIAN A (JKE, JKP, JKPK, JPP dan JKM): 25 MARKAH

INSTRUCTION:

This section consists of TWO (2) structured questions. Answer ONE (1) question only.

ARAHAN:

Bahagian ini mengandungi DUA (2) soalan struktur. Jawab SATU (1) soalan sahaja.

QUESTION 1

SOALAN 1

CLO1
C2

a) Expand the following algebraic expressions by using the method as stated in the bracket :
Kembangkan ungkapan algebra yang berikut dengan menggunakan kaedah yang dinyatakan di dalam kurungan :

i. $(y^2 - 3x)^5$ [Binomial Theorem]

[4 marks]

[4 markah]

ii. $\left(4 - \frac{x}{5}\right)^4$ [Pascal's Triangle]

[4 marks]

[4 markah]

CLO1
C2

b) Find the coefficient of x^{-3} in the expansion of $\left(x^3 + \frac{5}{x^6}\right)^{14}$.

Dapatkan pekali bagi x^{-3} dari pengembangan ungkapan $\left(x^3 + \frac{5}{x^6}\right)^{14}$.

[6 marks]

[6 markah]

CLO1
C2c) Expand $\sqrt{9+2x}$ until the third term.*Kembangkan $\sqrt{9+2x}$ sehingga sebutan ketiga.*

[5 marks]

[5 markah]

CLO1
C3d) Expand $\frac{1}{(1+x)^4}$ until the fourth term and estimate $\frac{1}{1.03^4}$ by using the expansion.Write the correct answer into **FOUR (4)** decimal places.*Kembangkan $\frac{1}{(1+x)^4}$ sehingga sebutan ke-empat dan anggarkan nilai bagi* *$\frac{1}{1.03^4}$ dengan menggunakan pengembangan tersebut. Tuliskan jawapan tepat kepada **EMPAT (4)** tempat perpuhan.*

[6 marks]

[6 markah]

QUESTION 2**SOALAN 2**CLO1
C3a) Expand the following functions until the first **FOUR (4)** terms.*Kembangkan fungsi berikut sehingga **EMPAT (4)** sebutan pertama.*

i. $e^{3x} + e^{-2x}$

[6 marks]

[6 markah]

ii. $\ln \frac{(1-4x)^2}{(1+3x)}$

[8 marks]

[8 markah]

CLO1
C3b) Find the Maclaurin Series of $\ln(3+5x)$ until the first **FOUR (4)** terms.*Dapatkan Siri Maclaurin bagi $\ln(3+5x)$ sehingga **EMPAT (4)** sebutan pertama.*

[11 marks]

[11 markah]

SULIT

SECTION B (JKE, JKP, JKPK, JPP and JKM): 25 MARKS

BAHAGIAN B (JKE, JKP, JKPK, JPP dan JKM): 25 MARKAH

INSTRUCTION:

This section consists of TWO (2) structured questions. Answer ONE (1) question only.

ARAHAN:

Bahagian ini mengandungi DUA (2) soalan berstruktur. Jawab SATU (1) soalan sahaja.

QUESTION 3

SOALAN 3

The position vectors of point A, B and C are $\overrightarrow{OA} = 3\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$, $\overrightarrow{OB} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\overrightarrow{OC} = -\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$. Determine;

Vektor kedudukan titik A, B dan C adalah $\overrightarrow{OA} = 3\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$, $\overrightarrow{OB} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ dan $\overrightarrow{OC} = -\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$. Tentukan;

CLO2
C1

a) \overrightarrow{AB}

[2 marks]

[2 markah]

CLO2
C3

b) $\overrightarrow{AB} \times \overrightarrow{BC}$.

[8 marks]

[8 markah]

CLO2
C2

c) $\overrightarrow{AB} \cdot \overrightarrow{BC}$

[3 marks]

[3 markah]

CLO2
C2

d) $|\overrightarrow{AC}|$

[4 marks]

[4 markah]

CLO2
C3

e) Angle between \overrightarrow{AB} and \overrightarrow{BC}

Sudut di antara \overrightarrow{AB} dan \overrightarrow{BC}

[8 marks]

[8 markah]

QUESTION 4

SOALAN 4

CLO2
C3

Express each of the following in partial fractions:

Ungkapkan pecahan separa di bawah:

(a) $\frac{3x-4}{x(x+1)}$

[5 marks]

[5 markah]

(b) $\frac{2x-4}{(1-2x)(1+x)}$

[5 marks]

[5 markah]

(c) $\frac{1}{(x-3)(x+1)^2}$

[8 marks]

[8 markah]

(d) $\frac{7x^2-18x-7}{(x-4)(2x^2-6x+3)}$

[7 marks]

[7 markah]

SULIT

SECTION C (JKE, JKP, JKPK) : 25 MARKS

BAHAGIAN C (JKE, JKP, JKPK) : 25 MARKAH

INSTRUCTION:

This section consists of TWO (2) structured questions. Answer ONE (1) question only.

ARAHAN:

Bahagian ini mengandungi DUA (2) soalan berstruktur. Jawab SATU (1) soalan sahaja.

QUESTION 5

SOALAN 5

CLO3
C3

- a) Verify the Laplace Transform of $f(t) = e^{3t}$ is $\frac{1}{s-3}$ by using the Definition method of

$$F(s) = \int_0^{\infty} e^{-st} f(t) dt.$$

Buktikan bahawa Jelmaan Laplace bagi $f(t) = e^{3t}$ adalah $\frac{1}{s-3}$ dengan menggunakan

kaedah Takrifan $F(s) = \int_0^{\infty} e^{-st} f(t) dt.$

[6 marks]

[6 markah]

CLO3
C3

- b) Determine the Laplace Transform for the following functions by using the Laplace Transform Table.

Tentukan Jelmaan Laplace bagi fungsi-fungsi yang berikut dengan menggunakan Jadual Jelmaan Laplace.

i. $f(t) = 10 + e^{4t} + 3e^{-t}$

[3 marks]

[3 markah]

ii. $f(t) = 2t + 9t^2 + t^2 e^{3t}$

[6 marks]

[6 markah]

iii. $f(t) = \sin\left(\frac{5}{7}t\right) + t\cos(4t) + 2\cos(t)$

[6 marks]

[6 markah]

CLO3
C3

c) Show the Laplace Transform of $e^{-2t} \cosh\left(\frac{t}{4}\right)$ by using the First Shifting Theorem.

Tunjukkan Jelmaan Laplace bagi $e^{-2t} \cosh\left(\frac{t}{4}\right)$ dengan menggunakan Teorem Anjakan

Pertama.

[4 marks]

[4 markah]

QUESTION 6

SOALAN 6

CLO3
C2

a) Find the Inverse Laplace Transform for each of the following functions.

Cari Jelmaan Laplace Songsang bagi setiap fungsi yang berikut.

i. $\frac{3}{s-4}$

[2 marks]

[2 markah]

ii. $\frac{5}{s-5} - \frac{4}{s^2-16}$

[3 marks]

[3 markah]

CLO3
C3

b) Find the Inverse Laplace Transform for the following expressions by using Partial Fraction method.

Dapatkan Jelmaan Laplace Songsang bagi ungkapan berikut menggunakan kaedah Pecahan Separa.

SULIT

i.
$$\frac{s}{(s+2)(s-4)}$$

[9 marks]

[9 markah]

ii.
$$\frac{2s^2 - 3}{s(s^2 + 1)}$$

[11 marks]

[11 markah]

SECTION D (JPP, JKM) : 25 MARKS

BAHAGIAN D (JPP, JKM) : 25 MARKAH

INSTRUCTION:

This section consists of **TWO (2)** structured questions. Answer **ONE (1)** question only.

ARAHAN:

Bahagian ini mengandungi **DUA (2)** soalan berstruktur. Jawab **SATU (1)** soalan sahaja.

QUESTION 7

SOALAN 7

CLO4
C2

a) Find the equation of the circle if :

Cari persamaan bulatan jika :

- i. AB is the diameter with the coordinates $A(-3,1)$ and $B(4,-2)$.
AB adalah garis pusat dengan koordinat $A(-3,1)$ dan $B(4,-2)$.

[3 marks]

[3 markah]

- ii. Centre $(-2,3)$, touching the y-axis.

Pusat $(-2,3)$, menyentuh paksi y.

[4 marks]

[4 markah]

CLO4
C3b) Calculate a length of tangent from the point $(3,-2)$ to the circle:

Kira panjang garis tangen dari titik $(3,-2)$ kepada bulatan:

$$2x^2 + 2y^2 + 8x - 14y - 26 = 0$$

[5 marks]

[5 markah]

SULIT

CLO4
C3

- c) Determine the focus, vertex and directrix then sketch the following curves of parabola:
Tentukan fokus, verteks dan direktriks kemudian lakarkan lengkungan parabola yang berikut :

i. $(x+3)^2 = 8y$

[6 marks]

[6 markah]

ii. $3y^2 + 12x - 3 = 0$

[7 marks]

[7 markah]

QUESTION 8

SOALAN 8

CLO4
C3

- a) Find the equation of the ellipse where the major vertices at $(4,2)$ and $(4,-4)$ and the foci at $(4,1)$ and $(4,-3)$

Dapatkan persamaan bagi persamaan elips yang mempunyai titik-titik vertex major $(4,2)$ dan $(4,-4)$ dan titik-titik fokus $(4,1)$ dan $(4,-3)$

[6 marks]

[6 markah]

CLO4
C3

- b) Find the coordinates of foci, vertices major, eccentricity and equations of directrices for ellipse equation $16x^2 + 9y^2 = 25$

Dapatkan titik-titik koordinat bagi fokus, vertex major, esentrik dan persamaan-persamaan direktriks bagi persamaan elips $16x^2 + 9y^2 = 25$.

[10 marks]

[10 markah]

CLO4
C3

- c) Find the coordinates of foci, vertices and equation of asymptotes for hyperbola equation $x^2 = 36 + 4y^2$

Dapatkan titik-titik koordinat fokus, vertex dan persamaan asimptot bagi persamaan hiperbola $x^2 = 36 + 4y^2$

[9 marks]

[9 markah]

SECTION E : 25 MARKS***BAHAGIAN E : 25 MARKAH*****INSTRUCTION:**

Answer **ONE (1)** question from section A, B or C (for **JKE, JKP** and **JKPK**) and section A, B or D (for **JKM, JPP**).

ARAHAN:

Jawab **SATU(1)** soalan dari bahagian A, B atau C (untuk **JKE, JKP** dan **JKPK**) dan bahagian A, B atau D (untuk **JKM, JPP**).

SOALAN TAMAT

FORMULA BA501

Binomial Expansion

1.	$(a+x)^n = a^n + {}^n C_1 a^{n-1} x + {}^n C_2 a^{n-2} x^2 + {}^n C_r a^{n-r} x^r + \dots$ (n = positive integer)
2.	$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \dots$ (n = negative integer or fraction)
3.	The (r+1) term, $T_{r+1} = {}^n C_r a^{n-r} x^r$

Power Series

1.	$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots + \frac{x^n}{n!}$
2.	$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + (-1)^{n-1} \frac{x^n}{n}$
3.	$\ln\left(\frac{m}{n}\right) = 2 \left[\frac{m-n}{m+n} + \frac{1}{3} \left(\frac{m-n}{m+n}\right)^3 + \frac{1}{5} \left(\frac{m-n}{m+n}\right)^5 + \dots \right]$
4.	$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \dots + \frac{f^n(0)x^n}{n!}$ (MACLAURIN)
5.	$f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)(x-x_0)^2}{2!} + \frac{f'''(x_0)(x-x_0)^3}{3!} + \dots + \frac{f^n(x_0)(x-x_0)^n}{n!}$ (TAYLOR)

Vector & Scalar

1.	Unit Vector, $\hat{u} = \frac{\bar{u}}{ u }$	2.	$\cos\theta = \frac{\bar{A} \cdot \bar{B}}{ A B }$	3.	$\vec{A} \cdot \vec{B} = a_1a_2 + b_1b_2 + c_1c_2$
4.	$\bar{A} \times \bar{B} = \begin{pmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{pmatrix}$	5.	Scalar triple product = Volume of parallelepiped $\equiv \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$	6.	Vector triple product $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b}(\mathbf{a} \cdot \mathbf{c}) - \mathbf{c}(\mathbf{a} \cdot \mathbf{b})$
7.	Area of parallelogram ABC $ \vec{AB} \times \vec{BC} $				

Laplace Transform

No	$f(t)$	$F(s)$	No.	$f(t)$	$F(s)$
1.	a	$\frac{a}{s}$	13.	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
2.	at	$\frac{a}{s^2}$	14.	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$
3.	t^n	$\frac{n!}{s^{n+1}}$	15.	$\sinh \omega t$	$\frac{\omega}{s^2 - \omega^2}$
4.	e^{at}	$\frac{1}{s-a}$	16.	$\cosh \omega t$	$\frac{s}{s^2 - \omega^2}$
5.	e^{-at}	$\frac{1}{s+a}$	17.	$e^{at} \sinh \omega t$	$\frac{\omega}{(s-a)^2 - \omega^2}$
6.	te^{-at}	$\frac{1}{(s+a)^2}$	18.	$e^{-at} \sinh \omega t$	$\frac{\omega}{(s+a)^2 - \omega^2}$
7.	$t^n \cdot e^{at}, n=1,2,3$	$\frac{n!}{(s-a)^{n+1}}$	19.	$e^{-at} \cosh \omega t$	$\frac{s+a}{(s+a)^2 - \omega^2}$

8.	$t^n \cdot f(t)$	$(-1)^n \frac{d^n}{ds^n} [F(s)]$	20.	$f_1(t) + f_2(t)$	$F_1(s) + F_2(s)$
9.	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$	21.	$\int_0^t f(u) du$	$\frac{F(s)}{s}$
10.	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$	22.	$f(t-a)u(t-a)$	$e^{-as} F(s)$
11.	$t \sin \omega t$	$\frac{2\omega s}{(s^2 + \omega^2)^2}$	23.	First derivative $\frac{dy}{dt}, y'(t)$	$sY(s) - y(0)$
12.	$t \cos \omega t$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$	24.	Second derivative $\frac{d^2 y}{dt^2}, y''(t)$	$s^2 Y(s) - sy(0) - y'(0)$

Non Linear Equation (Circle)

1.	$(x-a)^2 + (y-b)^2 = r^2$
2.	$x^2 + y^2 + 2gx + 2fy + c = 0$ $r = \sqrt{g^2 + f^2 - c}$ $center = (-g, -f)$
3.	Equation of a tangent and normal line, $y - y_1 = m(x - x_1)$ $m_{\text{tangent}} = -\frac{(x+g)}{(y+f)}$
4.	Radius, $r = \frac{ax + by + c}{\sqrt{(a)^2 + (b)^2}}$

PARABOLA

When the vertex lies at (0, 0) the standard equations for parabolas are:

Axis of symmetry	x - axis	y - axis
Description	opens right	opens up
Vertex	$(0, 0)$	$(0, 0)$
Focus	$(a, 0)$	$(0, a)$
Directrix	$x = -a$	$y = -a$
Equation	$y^2 = 4ax$	$x^2 = 4ay$

PARABOLA

When the vertex lies at (h, k) the standard equations for parabolas are:

Axis of symmetry	x -axis	y -axis
Description	opens right / left	opens up / down
Vertex	(h, k)	(h, k)
Focus	$(h + a, k)$	$(h, k + a)$
Directrix	$x = h - a$	$y = k - a$
Equation	$(y - k)^2 = 4a(x - h)$	$(x - h)^2 = 4a(y - k)$

ELLIPSE

The properties of the ellipse with center (0, 0) as follows :

Major axis	Parallel to x - axis		Parallel to y - axis	
Foci	$(c, 0)$	$(-c, 0)$	$(0, c)$	$(0, -c)$
Vertices	$(a, 0)$	$(-a, 0)$	$(0, a)$	$(0, -a)$
Equation	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $a > b > 0$ and $b^2 = a^2 - c^2$		$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ $a > b > 0$ and $b^2 = a^2 - c^2$	
Directrices	$x = \pm \frac{a^2}{c}$		$y = \pm \frac{a^2}{c}$	

ELLIPSE

The properties of the ellipse with center (h, k) as follows :

Major axis	Parallel to x - axis		Parallel to y - axis	
Foci	$(h + c, k)$	$(h - c, k)$	$(h, k + c)$	$(h, k - c)$
Vertices	$(h + a, k)$	$(h - a, k)$	$(h, k + a)$	$(h, k - a)$
Equation	$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$		$\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1$	
	$a > b$ and $b^2 = a^2 - c^2$		$a > b$ and $b^2 = a^2 - c^2$	
Directrices	$x = h \pm \frac{a^2}{c}$		$y = k \pm \frac{a^2}{c}$	

HYPERBOLA

The properties of the hyperbola with center $(0, 0)$ as follows :

Transverse axis	x - axis		y - axis	
Foci	$(c, 0)$	$(-c, 0)$	$(0, c)$	$(0, -c)$
Vertices	$(a, 0)$	$(-a, 0)$	$(0, a)$	$(0, -a)$
Equation	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$		$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$	
	$b^2 = c^2 - a^2$		$b^2 = c^2 - a^2$	
Asymptotes	$y = \frac{b}{a}x$	$y = -\frac{b}{a}x$	$y = \frac{a}{b}x$	$y = -\frac{a}{b}x$
Directrix	$x = \frac{a}{e}$	$x = -\frac{a}{e}$	$y = \frac{a}{e}$	$y = -\frac{a}{e}$

HYPERBOLA

The properties of the hyperbola with center (h, k) as follows :

Transverse axis	x - axis		y - axis	
Foci	$(h + c, k)$	$(h - c, k)$	$(h, k + c)$	$(h, k - c)$
Vertices	$(h + a, k)$	$(h - a, k)$	$(h, k + a)$	$(h, k - a)$
Equation	$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$		$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$	
	$b^2 = c^2 - a^2$		$b^2 = c^2 - a^2$	

Asymptotes	$y - k = \frac{b}{a}(x - h)$	$y - k = -\frac{b}{a}(x - h)$	$y - k = \frac{a}{b}(x - h)$	$y - k = -\frac{a}{b}(x - h)$
Directrix	$x = h + \frac{a}{e}$	$x = h - \frac{a}{e}$	$y = k + \frac{a}{e}$	$y = k - \frac{a}{e}$

Differentiation
$\frac{d}{dx}(k) = 0, k = \text{constant}$
$\frac{d}{dx}(x^n) = nx^{n-1}$
$\frac{d}{dx}(\ln u) = \frac{1}{u} \frac{du}{dx}$
$\frac{d}{dx}(e^u) = e^u \frac{du}{dx}$
$\frac{d}{dx}(\cos u) = -\sin u \frac{du}{dx}$
$\frac{d}{dx}(\sin u) = \cos u \frac{du}{dx}$
$\frac{d}{dx}(\tan u) = \sec^2 u \frac{du}{dx}$
$\frac{d}{dx}(\cot u) = -\text{cosec}^2 u \frac{du}{dx}$
$\frac{d}{dx}(\sec u) = \sec u \tan u \frac{du}{dx}$
$\frac{d}{dx}(\text{cosec } u) = -\text{cosec } u \cot u \frac{du}{dx}$

